Specifications for calculating several equations of relationship between two variables on a TYPE 650 ELECTRONIC COMPUTER

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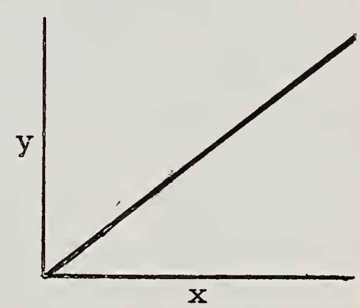
There are innumerable problems in forestry that require formulas expressing the relation between two measurements taken on each of a series of sampling units. An example is the familiar curve of tree height or tree volume over tree diameter. Many times the mathematical form of the relationship is unknown, and several possible forms must be investigated.

Countless hours have been spent developing these formulas with desk calculators or with even more primitive facilities. Now, because of modern electronic data-processing equipment, most of this time can be saved. The program described in this report calculates, at moderate cost, the following seven different equations of relationship between a dependent variable (y) and an independent variable (x):

(1) $y = bx$

where $b = \dfrac{\Sigma xy}{\Sigma x^2}$

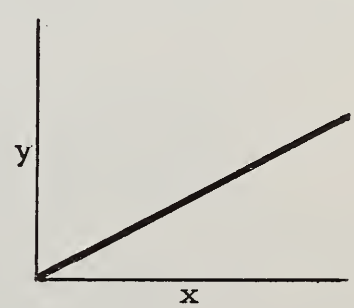
The curve form is a straight line characterized by the condition that when x is zero y will be zero.



(2) $y = bx$

where $b = \dfrac{\Sigma y}{\Sigma x}$

In this case, y is called a ratio estimate and b is called a ratio. The type of illustration in (1) above applies here also. In other words the curve is straight and passes through the origin.



(3) $y = bx^2$

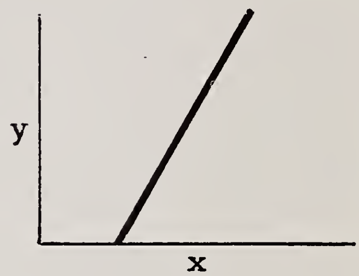
where $b = \dfrac{\Sigma x^2 y}{\Sigma x^4}$

This indicates a curve passing through the origin and with y varying directly with the square of x.

(4) $y = a+bx$

where $b = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$

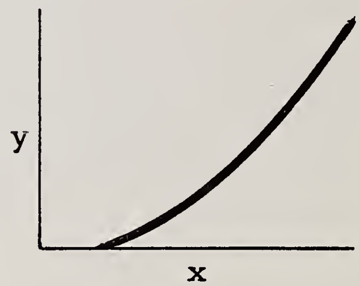
Here the curve form is a straight line which intercepts the y axis at "a" when x is zero. This is the so-called first-degree polynomial or linear least squares solution.



(5) $y = a+bx^2$

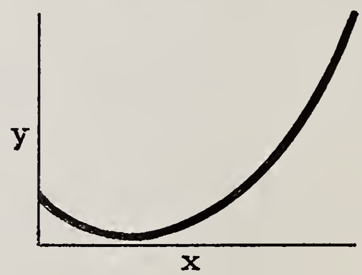
where $b = \dfrac{N\Sigma x^2 y - \Sigma x^2 \Sigma y}{N\Sigma x^4 - (\Sigma x^2)^2}$

This is a second-degree equation, with y varying directly as the square of x and intercepting the y axis at "a."



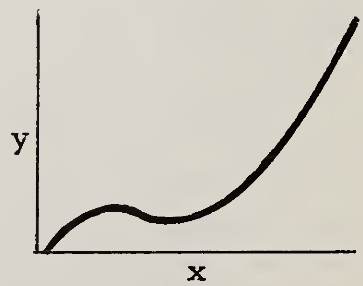
(6) $y = a+bx+cx^2$

where a, b, and c are constants for the second-degree polynomial or least squares quadratic solution.



(7) $y = a+bx+cx^2+dx^3$

where a, b, c, and d are constants for the third-degree polynomial or least squares cubic solution.



In addition, the program solves each of the seven equations for each of seven arbitrarily chosen values of the x variable. This is done

merely to facilitate plottings that will illustrate the relationships. The F test to determine if the explained variance due to the addition of a third or fourth constant is significant, will assist in choosing from among the three polynomial solutions (equations 4, 6, and 7).

All seven curve forms will be computed for every problem, even though some of these forms are known beforehand to be inappropriate. Time lost in these cases will be inconsequential.

INPUT SPECIFICATIONS

Input Card Type 000: Basic Data

One punched IBM card will accommodate a single pair of variables (y and x). These variables may be expressed either as whole numbers or as decimals. Either or both variables may be positive or negative. For cases where the variables are averages instead of basic elements of data, the number of basic elements associated with each pair of averages is also punched on the same card. This means that when basic elements of data (as opposed to averages) are used, the number "1" must be punched in the "frequency" columns of the card.

Provision was made for an additional weighting factor on input card type 000. This is to take care of situations in which variation in values of y for given values of x changes with values of x. It recognizes that the observations may be of differing worth. The code "1" is used for cases where there are no weights of this type. In fitting curves by least squares, it is assumed that the variance in y is the same for all values of x. When variance is not the same, the accepted procedure is to weight each observation by the reciprocal or inverse of its variance.[1/] These weighting factors are not appropriate for the ratio solution (equation 2) because variance is assumed proportional to x in that case. Thus when weighting factors are used, the ratio estimate will be meaningless.

1/ An example of the use of weighting factors is shown in a recent publication by Donald R. Gedney and Floyd A. Johnson, "Weighting Factors for Computing the Relation Between Tree Volume and D.B.H. in the Pacific Northwest." (U.S. Forest Serv. Pac. NW. Forest and Range Expt. Sta. Res. Note 174, 5 pp. 1959.)

Table 1 shows proper positions for this information on the punch card. All 80 card columns must be punched, either by some significant digit or by "0."

Table 1.--Card type 000: input specifications for basic data

Card column	Information
1 - 3	Card type (000).
4 - 10	Codes which identify the particular problem or set of data. These must be identical for all cards in a given problem but must be different for different problems.
11 - 20	y value: Negative values indicated by an "11" punch in the 20 or 30 column, or both.
21 - 30	x value: Negative values indicated by an "11" punch in the 20 or 30 column, or both.
31 - 40	Number of basic elements associated with y and x when they are averages, or with "1" when y and x are basic elements.
41 - 50	Weighting factor for y, such as inverse of variance, which is associated with the particular value of x in card column 21—30 or with "1" if there are no weights of this kind.
51 - 80	Other data or zeros (has no effect on program).

Input Card Type 001: Selected Values of x
For Plotting y Over x

As indicated earlier, the program solves each of the seven equations for each of seven arbitrarily chosen values of x, except for x equals zero. These values of x must be punched on input card type 001 (table 2).

Table 2.--Card type 001: input specifications for x plotting values

Card column	Information	
1 - 3	Card type (001).	
4 - 10	Either zeros or strata identification as desired (has no effect on program).	
11 - 20	1st selected x value	Negative values indicated by an "11" punch over columns 20, 30 ... 80.
21 - 30	2nd " " "	
31 - 40	3rd " " "	
41 - 50	4th " " "	
51 - 60	5th " " "	
61 - 70	6th " " "	
71 - 80	7th " " "	

Input Card Type 002: Decimal Point Positioning

One of the peculiarities of this program is that the electronic machine must be told where the decimal points lie. This is done by means of card type 002, specifications for which are given in table 3.

Table 3.--Card type 002: input specifications for decimal point positioning

Card column	Information
1 - 3	Card type (002).
4 - 10	Either zeros or strata identification as desired (has no effect on program).
11 - 12	Decimal point position code for y. (See table 4.)
13 - 20	1002 1002
21 - 22	Decimal point position code for x.
23 - 30	1003 1003
31 - 32	Decimal point position code for frequency.
33 - 40	1004 1004
41 - 42	Decimal point position code for weighting factors.
43 - 50	1005 1005
51 - 52	Decimal point position code for specific x values for which equation solutions are required.
53 - 60	1528 1534
61 - 80	Zeros.

If, for a particular project, the decimal points are on the extreme right of the 10-digit y values, for example, the code "10" must be entered in card columns 11 and 12. Similarly, if the decimal points are on the extreme left of the 10-digit inverse of variance values, the code "00" must be entered in card columns 41 and 42. A complete key to decimal point positioning codes is given in table 4.

Table 4.--Key to decimal point position input code

Decimal point position	Code
xxxxxxxxxx.	10
xxxxxxxxx.x	09
xxxxxxxx.xx	08
xxxxxxx.xxx	07
xxxxxx.xxxx	06
xxxxx.xxxxx	05
xxxx.xxxxxx	04
xxx.xxxxxxx	03
xx.xxxxxxxx	02
x.xxxxxxxxx	01
.xxxxxxxxxx	00

OUTPUT SPECIFICATIONS

For each set of input data cards, there will be 11 punched output cards, each identified by a card type number. Table 5 is a key to elements of information on these output cards. All elements of information except card type number and project identification codes are "floating point" numbers. This simply means that the first 2 digits of every 10-digit number are a code that indicates the decimal point position for the remaining 8-digit number. Table 6 is a key to these decimal point position codes.

Table 5.--Output specifications

				Card columns				
1 - 3[1/]	4 - 10[2/]	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80
010	xxxxxxx	b for $y = bx$ (least squares)	b for $y = bx$ (ratio)	b for $y = bx^2$ (least squares)	a for $y = a+bx$ (1st degree polynomial)	b for	a for $y = a+bx^2$ (least squares)	b for
020	xxxxxxx	a for $y = a+bx+cx^2$ (2nd degree polynomial)	b for	c for	a for $y = a+bx+cx^2+dx^3$ (3rd degree polynomial)	b for	c for	d for
030	xxxxxxx	F test for testing 1st degree polynomial	degrees of freedom for 1st degree test	F test for testing significance of 2nd degree polynomial over 1st degree	degrees of freedom for 2nd degree test	F test for testing significance of 3rd degree polynomial over 2nd degree polynomial	degrees of freedom for 3rd degree test	zeros
040	xxxxxxx	Solution of $y = bx$ (least squares) for values shown in output card 110						
050	xxxxxxx	Solution of $y = bx$ (ratio) for values shown in output card 110						
060	xxxxxxx	Solution of $y = bx^2$ (least squares) for values shown in output card 110						
070	xxxxxxx	Solution of $y = a+bx$ (1st degree poly.) for values shown in output card 110						
080	xxxxxxx	Solution of $y = a+bx^2$ (least squares) for values shown in output card 110						
090	xxxxxxx	Solution of $y = a+bx+cx^2$ (2nd degree poly.) for values shown in output card 110						
100	xxxxxxx	Solution of $y = a+bx+dx^3$ (3rd degree poly.) for values shown in output card 110						
110	xxxxxxx	Values of x used for calculating y in output cards 040 to 100						

1/ Card type.
2/ Problem identification.

Table 6.--Key to decimal point position output codes

First 2 digits of 10-digit numbers that appear on the 11 output card types	Last 8 digits of 10-digit numbers that appear on the 11 output card types
↑	↑
46	.000xxxxxxxx
47	.00xxxxxxxx
48	.0xxxxxxxx
49	.xxxxxxxx
50	x.xxxxxxx
51	xx.xxxxxx
52	xxx.xxxxx
53	xxxx.xxxx
54	xxxxx.xxx
↓	↓

Note: Decimal point positioning beyond the range shown can be obtained by extrapolation.

PANEL WIRING SPECIFICATIONS

533 Panel Wiring

Standard 80-80 panel.

407 Panel Wiring

Standard 80-column list.

TIME REQUIREMENTS

The condensed program deck is read into the 650 computer in 2 minutes. Each input card requires 2 seconds of machine time, and 60 seconds are required for final-phase calculations and for punching output cards.

OPERATING INSTRUCTIONS

1. Place cards in proper sequence

(a) Program deck.[2]	204 cards
(b) Plotting value card (input card type 001).	1 card
(c) Decimal point position card (input card type 002).	1 card
(d) Input data cards (input card type 000--must be a minimum of 5 cards).	variable

Repeat (b), (c), and (d) as many times as there are projects. However, (b) and (c) need not be repeated if the x plotting values and decimal point positioning are the same.

(e) Trailer card (80 zeros).	1 card

When changes occur in x plotting values, decimal point positioning, or in both, corresponding new input card types 001 and 002 must be inserted ahead of the input data cards.

2. Insert 533 panel.

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[2] This program deck may be obtained from the Pacific Northwest Forest and Range Expt. Sta., P. O. Box 4059, Portland 8, Oreg.

3. Set storage entry switches to: 70 - 1952 - 3000

4. Set program switch to: Stop

5. Set half-cycle switch to: Run

6. Set display switch to: Program register

7. Set overflow switch to: Stop

8. Set error switch to: Stop

9. Place program deck and data in hopper.

10. Ready blank cards in punch hopper.

11. Depress computer reset button.

12. Depress program start.

EXAMPLE OF APPLICATION

Problem

Calculate equations of relationship for board-foot tree volume (y) over tree diameter in inches at breast height (x).

Data

One hundred and thirteen paired observations, y and x, from a sample of 113 ponderosa pine trees.

Weighting factors

Each pair of y and x values is from a single tree. Thus frequency weights will all be "1." Variance in tree volumes by individual diameter classes was determined from a supplementary study. Variance in volume for 12-inch trees, for example, was found to be 811.22, and the inverse of variance weighting factor was therefore 0.0012327112.

Input information

A typical set of information for each of the three input cards follows.

Card Type 000 (See Table 1)

Card column	Code	Description
1 - 3	000	Card type
4 - 10	0001411	Baker County = 14 Ponderosa pine = 11
11 - 20	0000000073.	Tree volume
21 - 30	000000012.1	Tree diameter
31 - 40	0000000001	Frequency
41 - 50	0.0012327112	Inverse of variance
51 - 80	all zeros	

Card Type 001 (See Table 2)

Card column	Code	Description
1 - 3	001	Card type
4 - 10	0001411	Identification
11 - 20	0000000011	1st selected x value
21 - 30	0000000015	2nd selected x value
31 - 40	0000000020	3rd selected x value
41 - 50	0000000025	4th selected x value
51 - 60	0000000030	5th selected x value
61 - 70	0000000035	6th selected x value
71 - 80	0000000040	7th selected x value

Card Type 002 (See Table 3)

Card column	Code	Description
1 - 3	002	Card type
4 - 10	0001411	Identification
11 - 20	1010021002	Decimal point code for y in leading two digits
21 - 30	0910031003	Decimal point code for x in leading two digits
31 - 40	1010041004	Decimal point code for frequency in leading two digits
41 - 50	0010051005	Decimal point code for inverse of variance in leading two digits
51 - 60	1015281534	Decimal point code for x values for plotting in leading two digits
61 - 80	all zeros	

Output information

From output card types 010 and 020 (fig. 1), the seven equations of relationship described earlier were found to be:

(1) $y = 8.676x$

(2) $y = 7.810x$

(3) $y = 0.687x^2$

(4) $y = -393.858+38.462x$

(5) $y = -107.935+1.226x^2$

(6) $y = 155.980-34.138x+2.254x^2$

(7) $y = 152.472-33.514x+2.220x^2+0.00060x^3$

Output card type 030 indicates that the first-degree polynomial equation is statistically significant. Tabular F for the 1-percent level of probability for 1 and 111 degrees of freedom is about 6.85, far less than the observed F of 578.43. There is also evidence that a curved line of relationship can be defended over a straight-line relationship, since F = 146.33 again exceeds tabular F. The F test indicates no superiority of the third-degree curve over the second-degree equation.

After considering all seven possible solutions, the second-degree polynomial was tentatively accepted as most appropriate. Values for tree volume in card type 090 were then plotted over corresponding values of tree diameter in card type 110 to check the second-degree relationship for general reasonableness. For example, any particular equation could give either a minus volume or an impossibly large volume, say for 12-inch trees, and this might influence the final choice of an equation.

0101126801	5086761847	5078098383	4968655196	5239385809-	5138462361	5210793503-	5012259101
0201126801	5215597952	5134137660-	5022542939	5215247225	5133514255-	5022195024	4660237970
0301126801	5257842627	5211100000	5214633316	5211000000	4659183195	5210900000	0000000000-
0401126801	5195438031	5213014277	5217352369	5221690461	5226028554	5230366646	5234704738
0501126801	5185908221	5211714757	5215619676	5219524595	5223429514	5227334434	5231239353
0601126801	5183072787	5215447419	5227462078	5242909497	5261789676	5284102615	5310984831
0701126801	5129227881	5218307732	5237538913	5256770093	5276001274	5295232454	5311446363
0801126801	5140400092	5216789474	5238242901	5265825878	5299538406	5313938048	5318535211
0901126801	5153234821	5215113074	5237494388	5271147170	5311607142	5317226715	5323973434
1001126801	5153177007	5215117950	5237480714	5271121706	5311608609	5317241908	5324016582
1101126801	5111000000	5115000000	5120000000	5125000000	5130000000	5135000000	5140000000

Figure 1.--Output information for the example of application (see table 5 for key).